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Improving impact-echo method by using cross-spectral density

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Abstract

When the impact-echo method is used to detect internal flaws in materials and delaminations between layers, the signal which is collected by the transducer after rebounding in any interface is not always amenable to easy interpretation. The longitudinal waveform produced by multiple longitudinal wave reflections between two surfaces is affected by the interferences with the other waves generated by reflections (PS, SP and S waves) or with the Rayleigh wave and its rebounds from the borders of the material. This study applies signal processing techniques on the signals obtained, with finite elements simulation, in different points on the surface of a material. First, windowing is used to remove the Rayleigh wave, and after that, a new function called multicross-spectral density is defined applying cross-spectral density over the signals collected from points placed at several distances from the impact point. Two cases have been studied: calculation of the thickness of a concrete plate, and detection of a shallow crack (delamination) in the interface of a concrete slab with an asphalt overlay. The noise and the non-desired peaks in the frequency domain are reduced after the technique is applied facilitating the interpretation of results.

1. Introduction

The impact-echo method uses impact-generated stress waves that propagate in media to determine thickness or detect internal flaws in materials and delaminations between layers of different materials. The low-frequency waves propagate into the structure and are reflected by the external surfaces or by flaws. All the displacements versus time are recorded in a transducer located on the impact surface and are transformed to the frequency domain. Multiple reflections are identified in the spectrum and used to evaluate the structure. This method can be used in heterogeneous media (concrete, asphalt) as an advantageous alternative to ultrasonic methods because it avoids the problem of developing a transducer which generates low-frequency and short duration pulses with enough energy to penetrate concrete [1].

Many times the frequency spectrum contains several peaks which cannot be easily interpreted. When a longitudinal wave strikes a boundary an shear wave can also be produced. Likewise, an incident shear wave produces a longitudinal wave. So, many waves are travelling between the two free surfaces in a plate, due to the multiple reflections produced, and interferences occur [2]. This problem is even more complex when stress waves are reflected by a flaw, because the diffraction at the edges of the crack generates cylindrical wavefronts.

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Displacement waveforms represent the superposition of displacements caused by reflected and diffracted waves [3], and consequently, the frequency spectrum contains a large number of peaks. In addition the Rayleigh wave and the waves generated by its rebounding from the borders of the material also interfere with the longitudinal wave. Experience has shown that knowledge of the structure is very important to obtain successful results. Numerical studies provide an indispensable guide for the interpretation of experimental results and the understanding of the parameters that affect the results of the test [4].

Different techniques have been used to facilitate the interpretation of impact-echo signals. Already in the early works, it is recommended to saturate the Rayleigh wave and to cut the beginning of the signal [5]. As a result, the frequencies contained in the large amplitude Rayleigh wave are removed and the spectral response is clarified. Studying flaws, Abraham et al. [6] have applied a technique called time frequency analysis in which a temporal window $K(t)$ is previously used to overcome the attenuation of the signal after the Rayleigh wave; after that, the use of a coefficient of anomaly is proposed to express the void-related dimension. The wavelet transform has also been used [7] in an attempt to clarify the frequency spectrum of a concrete cylinder using a continuous wavelet transform (CWT) of the signal. The bispectrum method in conjunction with neural networks [8] have been employed for the detection and classification of concrete flaws. The cross-power spectrum and the coherence function has been used over surface waves instead of longitudinal waves [9] to evaluate the quality of concrete. The cross-correlation technique has also been used to detect small voids [10].

In this work, the Rayleigh wave is removed using a temporal window to increase the resolution of the longitudinal wave signal. However, after the removal, several peaks remain in the spectrum caused by the interferences between the different waves which propagate in the material. Furthermore these peaks are not at the same frequencies when the distances from the impact point are different. Nevertheless if the cross-spectral density function is applied over the spectra collected in different points, these peaks decrease and the frequency peaks which show the rebounds of the longitudinal wave are amplified. Repeating this procedure over several points, a spectrum with a few significant peaks is obtained. The influence of the number of samples which are collected is also considered in this study.

2. Numerical modelling

Modelling has been performed with the finite-elements software Ansys® and a Pentium® 4 computer with 512 MB RAM and 2.4 GHz has been used. The equations of motion are directly integrated step by step over time. Their solution is based on the Newmark direct integration scheme (implicit method). A frontal solver is used for solving the linear system of equations at each time step. In order to do a three-dimensional study with a minimum time of computation, a two-dimensional model is developed using rectangular quadratic axisymmetric elements.

In the impact-echo method, a transient stress pulse is introduced by the elastic impact of an object (a sphere) on the free surface. It can be represented by a half-sine curve. The duration of impacts (or contact time, t_c) depends on the radius of the sphere. In our study, $t_c = 14\,\mu s$ was used. The distribution of amplitudes and frequencies produced by the impact can be obtained by the Fourier transform of the half-sine curve. Experience shows that above $1.5/t_c$, approximately, (107 kHz in our case), the amplitudes of stress waves are not useful [11]. The impact region is 4 mm radius, simulated by applying a pressure load over several elements on the material surface, on the axis of symmetry. The maximum frequency, $f_{\max}$, which must be considered depends on how the wave is reflected by flaws or surfaces in the material. For a wave propagating at c_p through the material and being reflected at a distance d from the surface, the theoretical arrival frequency at the surface is

$$f_t = \frac{c_p}{2d}, \tag{1}$$

although the real frequency is given by $f_p = \beta f_t$ where β is a geometry-dependent coefficient (0.96 in plates) [1]. As detailed in Section 4.1, the through-thickness frequency in our experiments is 30.5 kHz and the highest frequency, due to the multiple longitudinal waves reflections between the delamination and the impact surface, is 64.5 kHz. The highest frequencies in flexural modes lie below 20 kHz. The sphere used in the experiment, with a maximum frequency content of 107 kHz, is enough to excite all these frequencies.

The sampling period, Δt_c, to record the time–displacement waveform is obtained from the Nyquist frequency, i.e., the double of the highest frequency considered (64.5 kHz),

$$\Delta t_c = \frac{1}{2f_{\max}} = 7.75\,\mu\text{s}. \tag{2}$$

However, for the sake of accuracy, sampling intervals that provide several samples per cycle are recommended, and a 1 μs sampling is used in this case.

To apply fast Fourier transform (FFT), 1024 points have been recorded. The recording time $T_{\max}$ is 1.024 ms and the resolution in the spectrum is $\Delta f = 1/T_{\max} = 0.9766$ kHz. Care must be taken to ensure that the grid spacing is correctly sized as to observe the effects of wave propagation. The recommended element size using an implicit method is

$$\Delta x = \lambda_{\min}/20, \tag{3}$$

where $\lambda_{\min}$ is the shortest wavelength along wave direction. The shortest wavelength can be calculated from the maximum frequency, $f_{\max}$, and the longitudinal wave velocity, c_p, as

$$\lambda_{\min} = c_p/f_{\max}, \tag{4}$$

whereby the grid spacing is established. The expected highest frequency is 64.5 kHz and the lower longitudinal wave velocity is 2635 m s^{-1} (asphalt). Therefore, using Eqs. (3) and (4), a 2 mm grid spacing is required.

3. Analysis procedure

The signals generated by the impact are collected in several points on the surface at different distances from the impact point. The collected time domain data is treated in the time domain and processed to the frequency domain using Matlab®. The collected time data is windowed to remove the Rayleigh wave. The temporal window used is a Blackman–Harris window [12]. It has slightly wider central lobes and its maximum sidelobes are minimized resulting in less sideband leakage than equivalent length Hamming and Hanning windows. The time t_r required by the Rayleigh wave to travel from the impact point to the measurement point is calculated as

$$t_r = d_m/c_r, \tag{5}$$

where d_m is the distance between the impact and the measurement point, and c_r the Rayleigh wave velocity given by

$$c_r = \frac{c_s}{1.14418 - 0.25771v + 0.12661v^2}, \tag{6}$$

where c_s denotes the shear wave velocity and v the Poisson ratio. The Blackman–Harris window is delayed by the travel time t_r and applied over the sampled signal. The equation for computing the coefficients of a minimum 4-term Blackman–Harris window with n samples is

$$w[k+1] = a_0 - a_1\cos\left(2\pi\frac{k}{n-1}\right) + a_2\cos\left(4\pi\frac{k}{n-1}\right) - a_3\cos\left(6\pi\frac{k}{n-1}\right), \tag{7}$$

where $0 \leqslant k \leqslant (n-1)$ and $a_0 = 0.35875, a_1 = 0.48829, a_2 = 0.14128, a_3 = 0.01168$. This Blackman–Harris window eliminates many points at the beginning and at the end of the signal, causing information about the first through-thickness reflections to be lost. This loss is avoided by modifying the window in such a way that the window applied to a record with n_p points has $n_p + n_p/10$ samples. Note that only the central n_p points are used in the calculations.

After windowing is applied, the cross-spectral density S_{xy} is calculated over each two signals x and y. S_{xy} can be related to their cross-correlation function R_{xy} through the Fourier transform relation

$$S_{xy}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xy}(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau, \tag{8}$$

considering the whole time and frequency range. Because the signal lengths are finite, the cross-spectral density of two signals x and y of equal length can be estimated as the product of the Fourier transform of x, $X(\omega)$, and the conjugate of the Fourier transform of y, $\bar{Y}(\omega)$:

$$S_{xy}(\omega) = X(\omega)\bar{Y}(\omega). \tag{9}$$

The cross-correlation and the cross-spectral density functions have been used to study problems in machinery [13]. From this it can be seen that when the system being investigated is strongly frequency dependent, the cross-spectral density function will be a relatively easily interpretable "peaked" curve; when the system is strongly time dependent (frequency independent), the cross-correlation function is peaked. If, as is supposed, our problem is strongly frequency dependent, the cross-spectral density will be readily interpreted. Looking at Eq. (9), only the frequency values with high amplitudes in both spectra will have high amplitude in S_{xy}. If this idea is applied to n signals $x_i(t)$ with Fourier transforms $X_i(\omega)$, a frequency signal $S_n(\omega)$ can be obtained as

$$S_n(\omega) = \prod_{i=1}^{n} |X_i(\omega)|, \tag{10}$$

which could be called *multicross-spectral density*. Only the frequency values with high amplitudes in all spectra, that is, the frequencies related to the response of the structure (thickness, delaminations, cracks) will have a high amplitude in S_n.

4. Experimental results

4.1. Response of a two-layer plate structure

First, a small plate made up of two layers is considered. A cylinder in which the height is very small with respect to the radius is used for applying axisymmetric calculations. The radius is 400 mm. The top layer (asphalt) has a thickness $d_1 = 20\,\text{mm}$, mass density $\rho_1 = 2280\,\text{kg}\,\text{m}^{-3}$ and longitudinal wave velocity $c_{p1} = 2635\,\text{m}\,\text{s}^{-1}$. The bottom layer has, respectively, $d_2 = 30\,\text{mm}$, $\rho_2 = 2400\,\text{kg}\,\text{m}^{-3}$ and $c_{p2} = 3687\,\text{m}\,\text{s}^{-1}$. Their acoustic impedances are calculated by $z_1 = \rho_1 c_{p1} = 6\,324\,000\,\text{kg}\,\text{m}^{-2}\,\text{s}^{-1}$ y $z_2 = \rho_2 c_{p2} = 8\,404\,080\,\text{kg}\,\text{m}^{-2}\,\text{s}^{-1}$. The waveform is more complex than for a simple plate and multiple frequencies of longitudinal wave reflections can appear, depending on the values of the reflection coefficient C_γ and the transmission coefficient C_t at the interface [14]:

$$C_\gamma = \frac{z_2 - z_1}{z_2 + z_1}, \tag{11}$$

$$C_t = \frac{2z_2}{z_2 + z_1}. \tag{12}$$

A complete study with regard to what frequencies can and cannot appear can be found in Ref. [15]. The relation between z_1 and z_2 influences the frequencies that do or do not appear. In this case, $z_1 < z_2$, and then, the theoretical frequency of the longitudinal waves reflected between the top surface and the interface is given by

$$f_i = \frac{c_{p1}}{4d_1} = 32.9\,\text{kHz}. \tag{13}$$

The theoretical frequency, with a coefficient $\beta = 0.96$ multiplying c_{pi}, of the longitudinal waves reflected between top and bottom surfaces is given by

$$f_c = \frac{1}{\dfrac{2d_1}{\beta c_{p1}} + \dfrac{2d_2}{\beta c_{p2}}} = 30.5\,\text{kHz}. \tag{14}$$

However, the displacements caused by longitudinal wave reflections from the interface are not significant when the coefficient C_γ is small (lower than 0.24). For the concrete slab and asphalt overlay in our experiment, the coefficient of reflection is 0.14 and the coefficient of transmission 1.14. Therefore, the response of the structure is dominated by the full thickness response $f_c = 30.5\,\text{kHz}$.

The shortest wavelength can be calculated from the expected maximum frequency, f_c, and the longitudinal wave velocity, c_{p1}, using Eqs. (3) and (4), resulting, approximately, in $\Delta x = 4.3\,\text{mm}$. A mesh with 2 mm spacing is deemed to be sufficient for data collection for this study. However, three different levels of mesh refinement are tested to show that the results are mesh independent: 0.5, 1 and 2 mm. For 0.5 mm grid spacing, 80 901 nodes are used, with a computation time of 7 h and 42 min. For 1 mm grid spacing, the number of nodes is 20 450 and the resulting time 1 h and 23 min while for 2 mm grid spacing those values are 6006 nodes and only 17 min, respectively. The data was recorded from 10 to 50 mm (20% and 100% of the thickness), at 2 mm intervals.

Fig. 1 shows the *Y*-direction displacement recorded at a surface point, 10 mm from the point of impact, using a 2 mm grid spacing. The Blackman–Harris window, showed as a dashed line in Fig. 1, is applied to the whole wave. This window is delayed 6.7 μs, time required by the Rayleigh wave to travel 10 mm. The window is defined with 1126 samples, although only the central 1024 points are used. The result is shown as the bold dark line in Fig. 1, from which it can be seen that the Rayleigh wave (the first large downward displacement) has been removed.

Fig. 2 shows the spectra in points 10 mm and 18 mm from the impact zone. In this spectra, windowing has been applied. It can be seen that the results for the different grid spacings are nearly identical. In both figures, a sharp peak appears at the frequency line 30.3 kHz, corresponding to the full thickness response. In addition several peaks appear at lower frequencies, corresponding with the natural frequencies of the plate. However, several less defined peaks occur at higher frequencies (between 35 and 55 kHz), and can be seen to be different in both points. This phenomena has been detected in all the recorded spectra. These peaks appear due to interference between the several waves appearing by reflection at the bottom and the top of the plate. The interpretation of the spectra is then not clear.

Applying Eq. (10) to several points between 10 and 18 mm, the multicross-spectral density (Fig. 3(a)) is obtained. The curves for a 0.5, 1 and 2 mm grid spacing are undistinguishable at the scale of the figure. Only one sharp peak placed at 30.3 kHz appears, with secondary peaks placed at low frequencies, which is due to the fact that the stress waves generated by the impact excite flexural modes of vibration, usually with amplitudes very large relative to the through-thickness frequency. A numerical study on the flexural modes of vibration of the plate is carried out using Ansys® software, with a level of mesh refinement equal to 2 mm. The natural frequencies of the model are extracted via a Block–Lanczos method. In this study the first six modes lie below 10 kHz. Therefore, a high-pass filter with a 10-kHz cutoff frequency is used with the purpose of removing these strong and low frequency components of the signal. Fig. 3(b) shows the results after the frequency filtering, where it should be noted that only the peak at 30.3 kHz (through-thickness peak) remains.

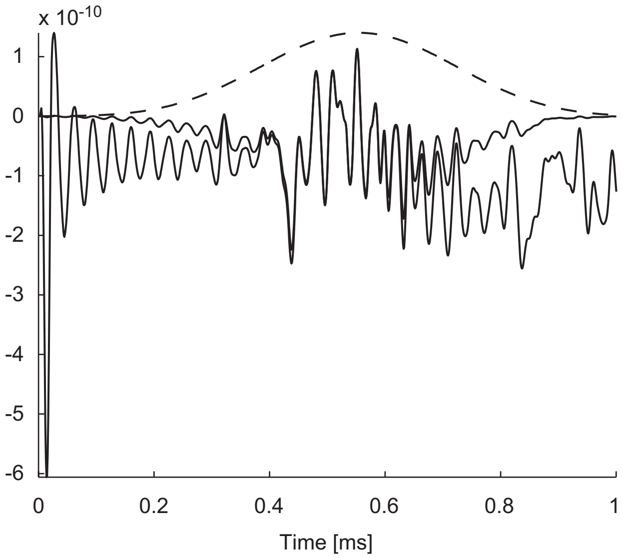

Fig. 1. Windowing effect over a recorded wave: solid line for the recorded wave, dashed line for the Blackman–Harris window and bold dark line for the windowed signal.

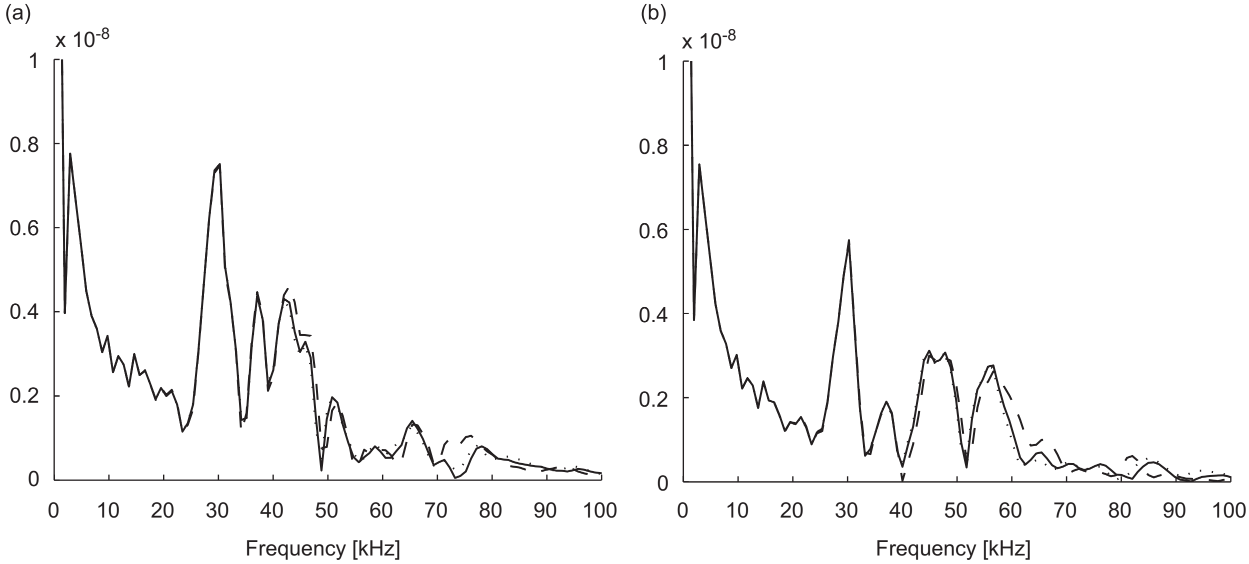

Fig. 2. Two-layer plate structure. Spectra at points from: (a) 10 mm; (b) 18 mm. The dotted line shows the results for 0.5 mm grid spacing whereas the solid and dashed lines correspond to grid spacing of 1 mm and 2 mm, respectively.

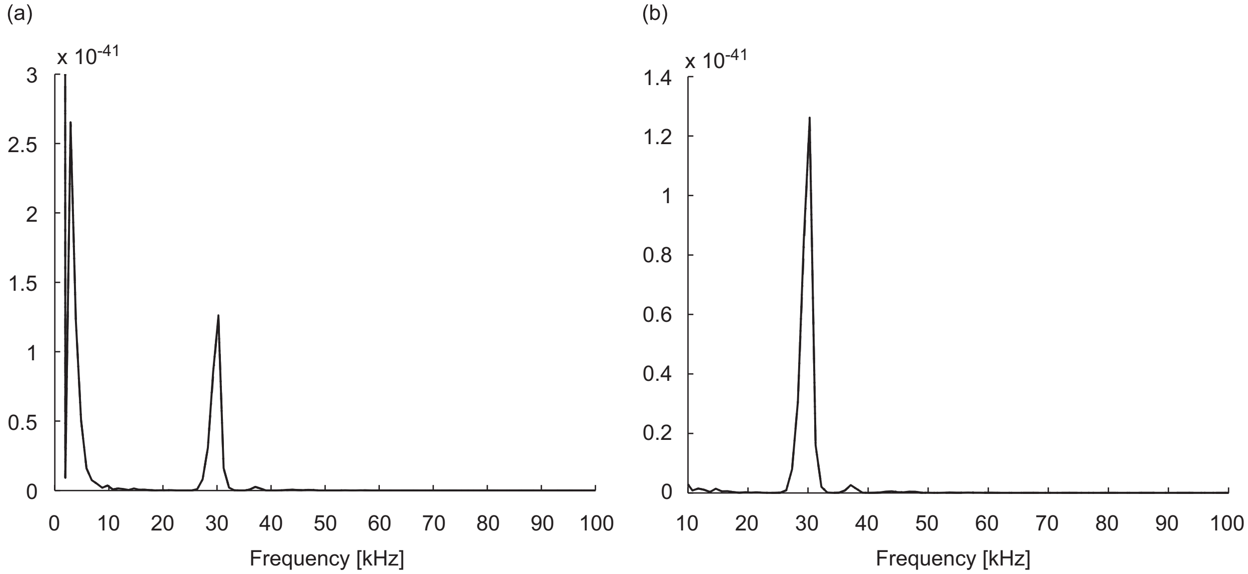

Fig. 3. Two-layer plate structure. Multicross-spectral density at points between 10 and 18 mm: (a) whole; (b) filtering low-frequencies.

Fig. 4 shows the averaged spectra corresponding to the same points between 10 and 18 mm as in Fig. 3. Although the main peaks also occur, several non-desired peaks remain at higher frequencies. Therefore, the multicross-spectral density is more suitable for a precise identification of the full thickness response of the two layer structure.

4.2. Response of a two-layer plate structure with delaminations

Now, the previous plate is considered again but with a shallow crack (delamination) in the contact zone between the concrete slab and the asphalt overlay. The stress waves generated by the impact can excite

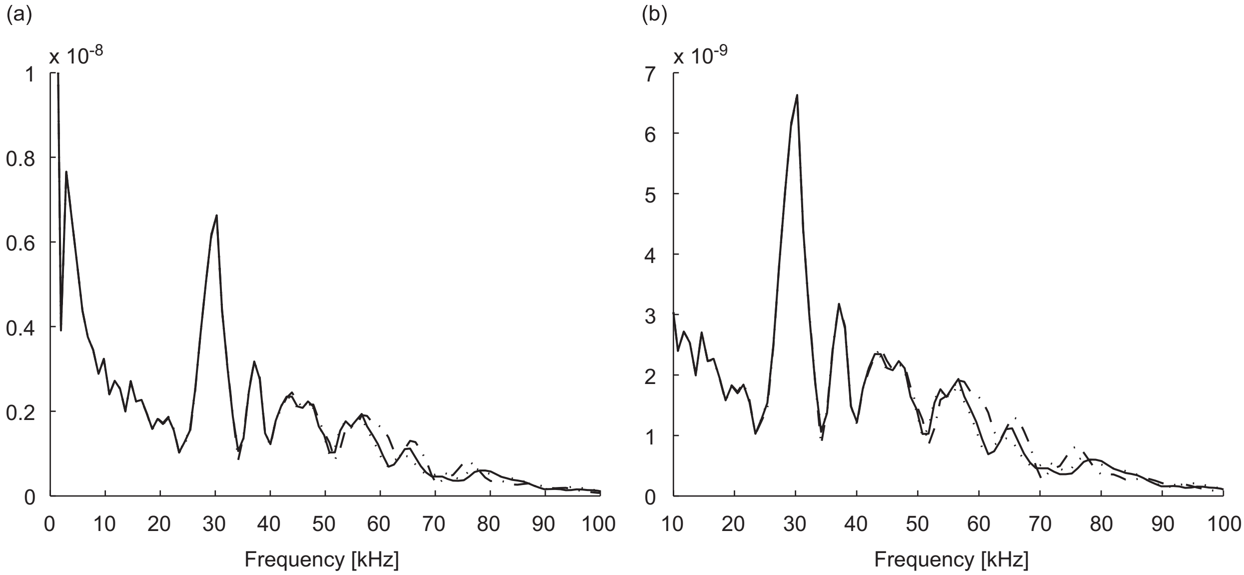

Fig. 4. Two-layer plate structure. Averaged spectra at points between 10 and 18 mm: (a) whole; (b) filtering low frequencies. The dotted line shows the results for 0.5 mm grid spacing whereas the solid and dashed lines correspond to grid spacing of 1 mm and 2 mm, respectively.

multiple flexural modes of the thin layer above the shallow crack. Two different components can be identified [11]: a lower frequency signal produced by the flexural vibration of the thin layer above the delamination and a higher frequency signal produced by multiple longitudinal waves reflections between the delamination and the impact surface, with a frequency

$$f_i = \frac{\beta c_{p1}}{2d_1} = 63.2\,\text{kHz} \tag{15}$$

as they would be at an air-asphalt interface ($\beta = 0.96$). The full thickness response of the structure ($f_c = 30.5$ kHz) and the flexural modes of vibration of the whole plate must be also considered.

The experiment is carried out for a 50 mm-radius delamination. The impact region has not been changed and a 2 mm grid spacing is used. Fig. 5(a) shows the spectra, after the Rayleigh wave has been removed using the Blackman–Harris window, at points 10 and 18 mm from the impact point. The spectra show a large sharp peak whose frequency is about 6 kHz. A frequency of 17.6 kHz also appears in the spectra. The full thickness frequency $f_c = 30.5$ kHz does not appear in the spectra. The small peak appearing in the spectra at 64.5 kHz, corresponds to the reflections between the impact surface and the top surface of the delamination. This frequency is often difficult to detect because the amplitudes of the flexural modes of vibration are very large relative to the higher frequency delamination vibration. Consequently, a numerical study on the flexural modes of vibration of the whole plate and of the thin layer above the shallow crack is required. Ansys® software is used with a level of mesh refinement equal to 2 mm. The Block Lanczos technique is used again for modal extraction. The study shows that the first flexural modes of the delamination take place at 6223 Hz (Fig. 6), along with another flexural mode at 18 080 Hz. All the main flexural modes lie below 20 kHz. Therefore, a high-pass filter with a cutoff frequency at 20 kHz can be used to remove the flexural frequencies. Fig. 5(b) shows the filtered spectra. A sharp peak placed at 64.5 kHz is now the main peak, although other secondary peaks also appear.

Applying Eq. (10) to points between 10 and 18 mm, the multicross-spectral density (Fig. 7) is obtained. In Fig. 7(a), without filtering, only a sharp peak appears, corresponding to the first flexural mode of the

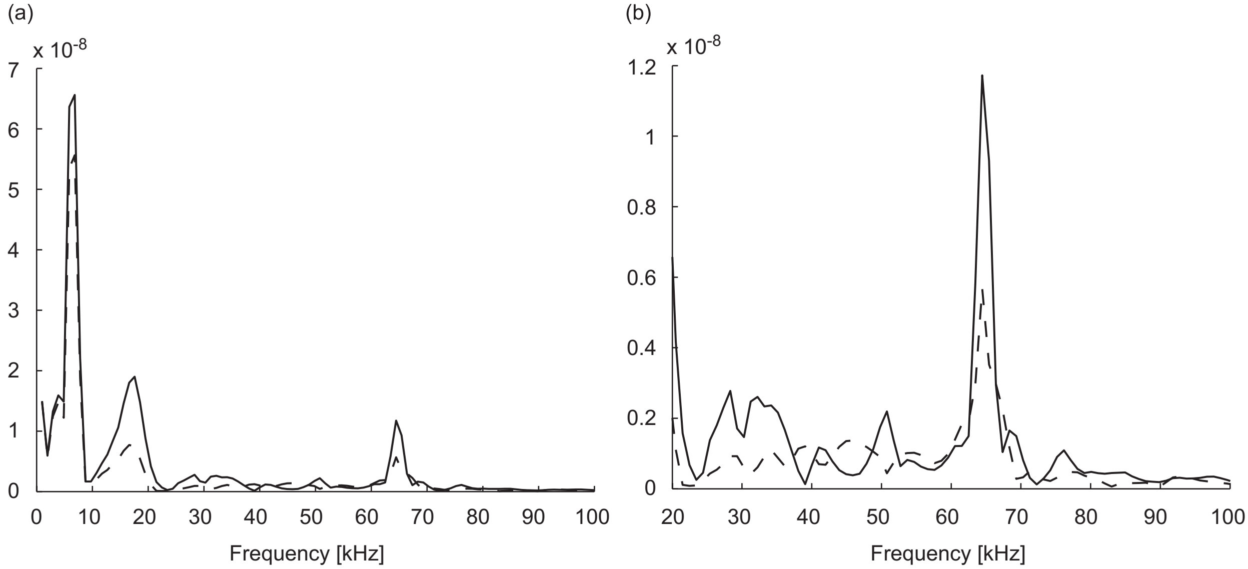

Fig. 5. Response of a shallow crack: (a) spectra at points from 10 mm (solid line) and 18 mm (dashed line); (b) filtered spectra.

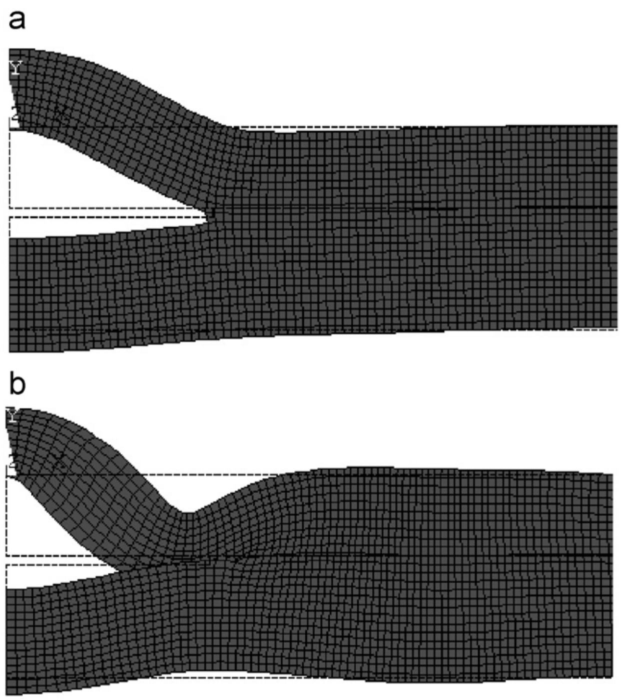

Fig. 6. Flexural modes of a plate with a shallow crack: (a) 6223 Hz; (b) 18 080 Hz.

delamination. When low frequencies are removed (Fig. 7(b)), only a peak at 64.5 kHz appears, produced by multiple longitudinal wave reflections between the impact surface and the delamination. The multicross-spectral density evidences the existence of a delamination. In this way, the depth at the delamination is located can be determined easily.

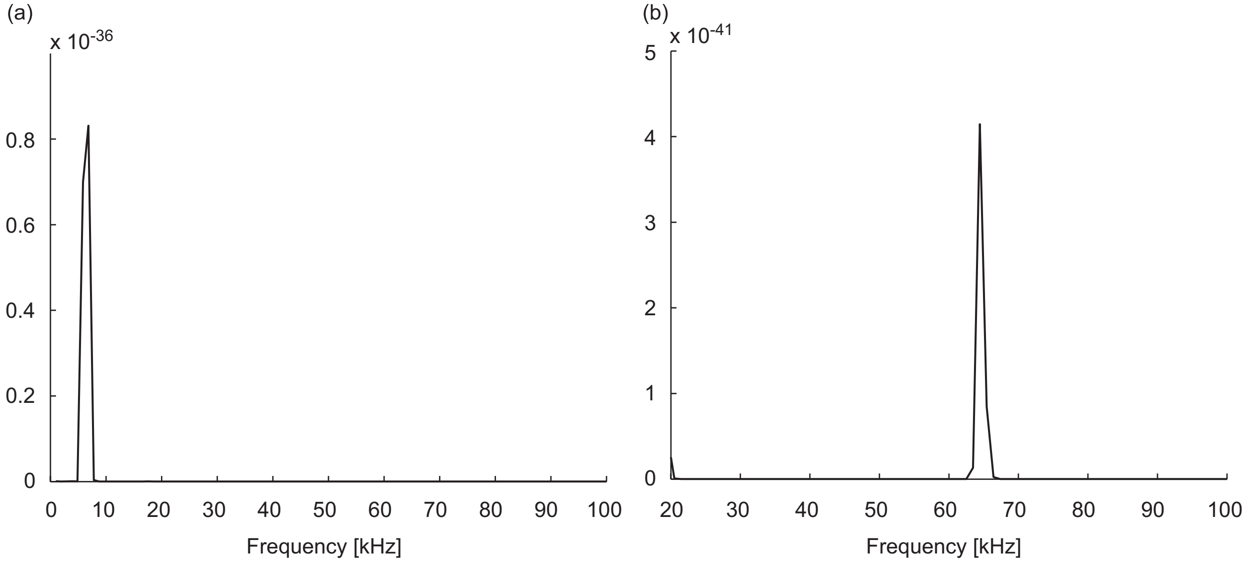

Fig. 7. Response of a shallow crack. Multicross-spectral density for points between 10 and 18 mm: (a) whole; (b) filtering low frequencies.

5. Conclusions

The impact-echo method is useful to measure the thickness of plates and to detect the delaminations between layers, but an appropriate signal processing is recommended to facilitate the interpretation of the results. The Rayleigh wave must be removed to avoid interferences and, in this paper, a Blackman–Harris window is satisfactorily applied for this purpose. To avoid eliminating too many points at the beginning and at the end of the signal, the window is widened and only the central points are used. Nevertheless, the interpretation of the results in the frequency domain is often difficult even if the Rayleigh wave has been removed. This work shows that the application of the cross-spectral density between two different surface points improves the results. Repeating this process at several points, a new function called multicross-spectral density is defined. This function multiplies the magnitude spectra recorded at several distances from the impact point, and reduces the noise and the non-desired peaks. Usually the resulting spectrum is easier to interpret, and the performance of the impact-echo method is improved. In this work the multicross-spectral density has been applied to determine the thickness of a two-layer plate structure (concrete with asphalt overlay), and to the determination of the depth of a delamination. In both cases the multicross-spectral density improves the results compared to direct application of FFT. When FFT is used, even if the spectra are averaged, several peaks at non-desired frequencies remain and the interpretation of results is not clear. The multicross-spectral density removes these peaks and only the flexural modes and the through-thickness frequencies remain. This work also shows that filtering the signal improves the technique when the amplitudes of the flexural modes are very large relative to the through-thickness frequency. A previous modal analysis should be performed to determine the frequencies of these flexural modes and a high-pass filter should be then designed to remove them. As a result of the application of the procedure described above, only the through-thickness frequencies will remain in the spectrum. The proposed multicross-spectral density technique provides very accurate results for the through-thickness frequencies as shown by the studies performed with different levels of mesh refinement and different filters.

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